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ONE METHOD OF CALCULATION OF THE HYDRAULIC FRACTURING OF A FORMATION

Kh. M. Gamzaev

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For determining the radius of propagation of a system of cracks formed in hydraulic fracturing of a formation, the equation of turbulent filtration of a fluid (breakdown agent) in the formation is used. A formula for evaluating the hydrodynamic efficiency of hydraulic fracturing of the formation is obtained.

Keywords: *hydraulic fracturing of the formation, turbulent filtration, moving boundary, method of successive change of steady states.*

Introduction. Hydraulic fracturing of a formation is one of the most popular technologies of extraction of oil from low-permeability collectors. To realize it one injects, into a permeable formation via a well, a breakdown agent under such pressure and flow rate sufficient to fracture the formation and to form cracks. Thereafter a proppant (artificial sand) is pumped into the well using a transferable working fluid with the aim of keeping the crack in an open state followed by isolation from the pressure and putting the well into production. Hydraulic fracturing of the formation is realized mainly in the following wells [1]:

(a) wells with a high formation pressure but a low permeability of the collector and with a contaminated critical area;
(b) those with a low efficiency;
(c) low-pickup injection wells;
(d) those with a high gas factor.

When the hydraulic fracturing of a formation is carried out, it is quite important to evaluate its hydrodynamic efficiency. For this purpose one traditionally uses the equation of oil influx to the well in the steady-state regime, where the effect from hydraulic fracturing is expressed by a negative value of the skin factor. The value of the skin factor is computed from the data on the geometry of the crack (length, width, height, and residual permeability of the proppant member). However, an analysis of the data of hydrodynamic investigations of wells shows that in hydraulic fracturing, the crack formed in the formation does not have a classical linear appearance and represents a few branches, i.e., a system of cracks (conjugated fractures) is actually formed in the formation to a certain distance from the well [2]. Consequently, to evaluate the hydrodynamic efficiency of hydraulic fracturing of a formation we must know the radius of propagation of the system of cracks in the formation, which is determined below using a mathematical model of turbulent filtration of a single-phase fluid in the formation.

Formulation of the Problem. Let us assume that a breakdown agent is injected into a horizontal homogeneous formation with a constant thickness H via a well of radius r_w; it is injected under such pressure and flow rate sufficient to fracture the rock and to form cracks. It can be assumed that turbulent filtration of the breakdown agent occurs in the formation; a mathematical model of the turbulent filtration can be represented in the following form:

$$c\frac{\partial P}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rU)=0\,, \tag{1}$$

$$-\frac{\partial P}{\partial r}=\frac{\mu}{k}U+\beta\rho U^2\,. \tag{2}$$

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It should be noted that formula (1) represents the equation of continuity of the plane-radial filtration flow of a homogeneous incompressible breakdown agent in a deformable porous medium, whereas (2) is the equation of motion written as a binomial filtration law [3]. The first term on its right-hand side allows for laminar effects; the second term allows for inertial turbulent effects in breakdown-agent flow in the formation. Clearly, for high filtration rates, the actions of the laminar effects can be disregarded. Then the system of equations (1)–(2) can be transformed to the form

$$\frac{\partial P}{\partial t}+\frac{1}{c\sqrt{\beta\rho}\,r}\frac{\partial}{\partial r}\left(r\sqrt{-\frac{\partial P}{\partial r}}\right)=0\,. \tag{3}$$

Let us assume that at the initial instant of time, the pressure throughout the formation is the same and equal to P_0. Then we will have, for Eq. (3), the initial condition

$$P\,(r,0)=P_0\,. \tag{4}$$

If the breakdown agent is injected into the well with volume flow rate Q = const, the boundary condition on the well can be written as

$$2\pi rHU\big|_{r=r_{\rm w}}=Q \quad \text{or} \quad \frac{2\pi Hr}{\sqrt{\beta\rho}}\sqrt{-\frac{\partial P}{\partial r}}\,\Bigg|_{r=r_{\rm w}}=Q\,. \tag{5}$$

Clearly, a disturbed region with a moving boundary is formed around the well on injection of the breakdown agent into it. Since the pressure is constant everywhere in an undisturbed region and is equal to the initial pressure, we will have, at the moving boundary of the disturbed region, the following conditions:

$$P\big|_{r=\xi(t)}=P_0\,, \tag{6}$$

$$\frac{\partial P}{\partial r}\bigg|_{r=\xi(t)}=0\,. \tag{7}$$

Relation (6) demonstrates the continuity of the pressure, whereas (7) points to the condition of smoothness of the pressure curve at the boundary of the disturbed region. It is necessary to note that the law of variation in the radius of the disturbed region with time is unknown.

Let us assume that injection of the breakdown agent into the well lasts for the period T. Then the problem of determination of the radius of propagation of the system of cracks formed in hydraulic fracturing of the formation is reduced to determination of $\xi(T)$, i.e., the radius of the disturbed region at the instant of time $t = T$, on the basis of solution of problem (3)–(7).

Method of Solution. To solve problem (3)–(7) we use the method of successive change of steady states [4]. In accordance with it, we assume that within the time t sec after the beginning of injection of the breakdown agent, a disturbed region of radius $\xi(t)$ is formed around the well; the pressure distribution in this region is determined as the solution of the steady-state equation

$$\frac{1}{c\sqrt{\beta\rho}\,r}\frac{\partial}{\partial r}\left(r\sqrt{-\frac{\partial P}{\partial r}}\right)=0\,. \tag{8}$$

The initial formation pressure P_0 is preserved in the remaining part of the formation. Integrating (8) with respect to r and allowing for conditions (5) and (6), we obtain

$$P(r,t)=\begin{cases}P_0+\dfrac{Q^2\beta\rho}{4\pi^2H^2}\left(\dfrac{1}{r}-\dfrac{1}{\xi(t)}\right), & r_w\le r\le\xi(t)\,;\\ P_0\,, & r\ge\xi(t)\,.\end{cases}\qquad(9)$$

To determine the law of variation in the radius of the disturbed region with time we integrate Eq. (3) with respect to the variable coordinate r on the segment $[r_w, \xi(t)]$. Integrating by parts and allowing for condition (7), we have

$$\int_{r_w}^{\xi(t)} r\frac{\partial P}{\partial r}dr-\frac{1}{c\sqrt{\beta\rho}}\,r\sqrt{-\frac{\partial P}{\partial r}}\,\Bigg|_{r=r_w}=0\,.$$

Substitution of the expression $P(r, t)$ from (9) into the last relation yields an ordinary differential equation for the radius of the disturbed region $\xi(t)$

$$\left(1-\frac{r_w^2}{\xi^2}\right)\frac{d\xi}{dt}=\frac{4\pi H}{cQ\beta\rho}\,.\qquad(10)$$

The initial condition for Eq. (10) can be written in the form

$$\xi(0)=r_w\,.\qquad(11)$$

Integration of Eq. (10) with account for initial condition (11) yields the law of time variation in the radius of the disturbed region

$$\xi(t)+\frac{r_w^2}{\xi(t)}=\frac{4\pi H}{cQ\beta\rho}\,t+2r_w\,.$$

Hence, disregarding r_w^2, we can determine the approximate value of the radius of propagation of the system of cracks in hydraulic fracturing of the formation

$$\xi(T)=\frac{4\pi H}{cQ\beta\rho}\,T+2r_w\,.\qquad(12)$$

Now, knowing the radius of propagation of the system of cracks in the formation, we can pass to determination of the hydrodynamic efficiency of hydraulic fracturing. Let us assume that the entire oil flowing to the well at a distance $r = \xi(T)$ arrives at the system of cracks and then moves in it up to the well's wall without resistance. This corresponds to the radial oil influx to a well with a radius equal to the radius of propagation of the system of cracks $\xi(T)$. Then for evaluation of the hydrodynamic efficiency of hydraulic fracturing of the formation, we can use the formula

$$\varphi=\frac{Q_T}{Q_0}\,.\qquad(13)$$

The quantities Q_0 and Q_T can be determined from Dupuy's formula:

$$Q_0=\frac{2\pi kH}{\mu_o}\frac{P_b-P_w}{\ln\dfrac{R}{r_w}}\,,\quad Q_T=\frac{2\pi kH}{\mu_o}\frac{P_b-P_w}{\ln\dfrac{R}{\xi(T)}}\,.$$

Substitution of the expressions of Q_0 and Q_T into (13) yields

$$\varphi = \frac{\ln \frac{R}{r_w}}{\ln \frac{R}{\xi(T)}} . \qquad (14)$$

Thus, formulas (12) and (14) enable us to find the radius of propagation of the system of cracks formed in hydraulic fracturing of the formation and to evaluate its hydrodynamic efficiency.

Conclusions. When the hydraulic fracturing of a formation is carried out in the well a system of cracks is formed around it. The radius of propagation of the system of cracks in the formation is determined on the basis of solution of the equation describing the breakdown-agent filtration in a turbulent regime. The formula for evaluating the hydrodynamic efficiency of hydraulic fracturing of the formation is proposed.

NOTATION

c, compressibility factor of the porous medium, Pa^{-1}; H, formation thickness, m; k, absolute permeability of the formation, m^2; P, pressure, MPa; P_0, initial pressure in the formation, MPa; P_b, pressure at the external boundary of the formation, MPa; P_w, well pressure, MPa; Q, volume flow rate of the breakdown agent, m^3/sec; Q_0, yield of a hydrodynamically perfect well before hydraulic fracturing, m^3/sec; Q_T, yield of the well after hydraulic fracturing, m^3/sec; r, radial coordinate, m; R, formation radius, m; r_w, well radius, m; t, time, sec; T, instant of time, sec; U, filtration rate of the agent, m/sec; β, turbulence factor, m^{-1}; μ, dynamic viscosity of the breakdown agent, Pa·sec; μ_0, oil viscosity, Pa·sec; $\xi(t)$, radius of the disturbed region, m; ρ, density of the breakdown agent, kg/m^3; φ, hydrodynamic efficiency of hydraulic fracturing of the formation. Subscripts: b, external boundary; 0, initial value; o, oil; w, well.

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